SOLUTION SET V

EXERCISE V.1 : RLC CIRCUIT IN SINUSOIDAL REGIME

The impedance of the serial RLC circuit has been established in Exercise II.1:

$$\underline{Z} = R + j\,\frac{\omega_k^2 LC - 1}{\omega_k C} \quad \text{with : } \omega_k = 2\,\pi\,f_k$$

The rms value of the current, I, and the power factor, cosϕ, are given by relationships:

$$I = \frac{U}{|\underline{Z}|} \qquad \text{and} \qquad \cos\phi = \cos(\arg(\underline{Z}))$$

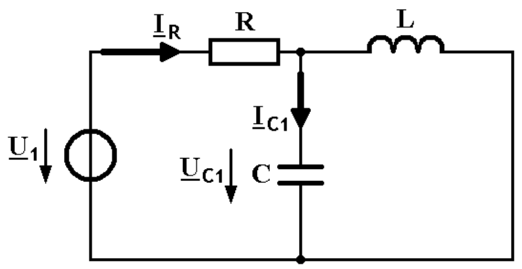
The results obtained for the two frequencies are summarized in the table below :

f [Hz]	$\lvert \underline{Z} \rvert$ [Ω]	arg($\underline{Z}$) [°]	I [A]	cosϕ	Nature of the impedance
1000	13,88	– 43,93	7,20	0,72	capacitive
2000	11,01	24,74	9,08	0,91	inductive

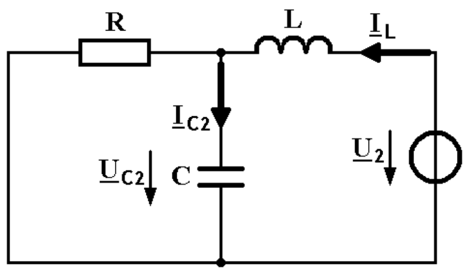
EXERCISE V.2 : SUPERPOSITION IN SINUSOIDAL REGIME

The superposition principle allows to break down the studied circuit into two circuits :

Circuit 1



Circuit 2



In circuit 1, the equivalent impedance $\underline{Z}_1$ is written (case 3 of Exercise II.1) :

$$\underline{Z}_1 = R + j\,\frac{\omega_1 L}{1 - \omega_1^2 LC}$$

Currents and voltages will be given by:

$$\underline{I}_R = \frac{\underline{U}_1}{\underline{Z}_1}\ ; \qquad \underline{U}_{C1} = \underline{U}_1 - R \cdot \underline{I}_R\ ; \qquad \underline{I}_{C1} = \underline{U}_{C1}\, j\,\omega_1 C$$

Numerical application :

$\underline{U}_1 = 100 \cdot e^{j90°}$ V ; $\underline{Z}_1 = 31{,}48 \cdot e^{j17{,}66°}$ Ω

$\Rightarrow \quad \underline{I}_R = 3{,}18 \cdot e^{j\,72{,}33°}$ A $= (0{,}964 + j\,3{,}03)$ A

$\Rightarrow \quad \underline{U}_R = R \cdot \underline{I}_R = 95{,}3 \cdot e^{j\,72{,}33°} \qquad \underline{U}_{C1} = \underline{U}_1 - \underline{U}_R = 30{,}34 \cdot e^{j\,162{,}34°}$ V

$\Rightarrow \quad \underline{I}_{C1} = \underline{U}_{C1} / \underline{Z}_{C1} = \underline{U}_{C1} \cdot j\omega_1 C = 2{,}86 \cdot e^{-j\,107{,}66°}$ A

In circuit 2, the equivalent impedance $\underline{Z}_2$ is written (case 4 of Exercise II.1) :

$$\underline{Z}_2 = \frac{R + j\,\omega_2(\omega_2^2 R^2 LC^2 + L - R^2 C)}{1 + (\omega_2 RC)^2}$$

Currents and voltages will be given by:

$$\underline{I}_L = \frac{\underline{U}_2}{\underline{Z}_2} \; ; \qquad \underline{U}_{C2} = \underline{U}_2 - j\,\omega_2 L \cdot \underline{I}_L \; ; \qquad \underline{I}_{C2} = \underline{U}_{C2}\, j\,\omega_2 C$$

Numerical application :

$\underline{U}_2 = 100 \cdot e^{j90°}$ V ; $\underline{Z}_2 = 4{,}99 \cdot e^{j\,79{,}50°}$ Ω

$\Rightarrow \quad \underline{I}_L = 20{,}03 \cdot e^{j\,10{,}50°}$ A $= (19{,}7 - j\,3{,}65)$ A ;
$\underline{U}_{C2} = (36{,}69 - j\,98{,}0)$ V $= 104{,}64 \cdot e^{j\,69{,}47°}$ V

$\Rightarrow \quad \underline{I}_{C2} = 19{,}72 \cdot e^{j\,20{,}53°}$ A

At this point, it is time to remember that we can not add two phasors of different frequencies: only the instantaneous values add up in such a case. We will write (with the angles in radians) :

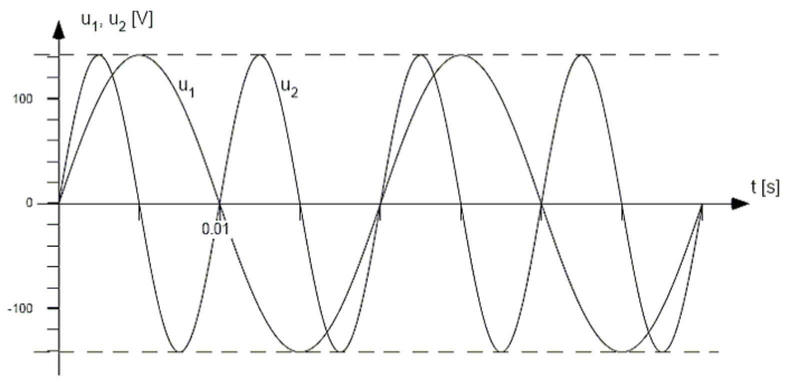
$$u_{c1}(t) = |\,\underline{U}_{C1}\,| \sqrt{2} \cos(\omega_1 \cdot t + \arg(\underline{U}_{C1})) = \underline{42{,}9 \cos(314{,}2 \cdot t + 2{,}827)\text{ V}}$$
$$u_{c2}(t) = |\,\underline{U}_{C2}\,| \sqrt{2} \cos(\omega_2 \cdot t + \arg(\underline{U}_{C2})) = \underline{148{,}0 \cos(628{,}3 \cdot t - 1{,}212)\text{ V}}$$

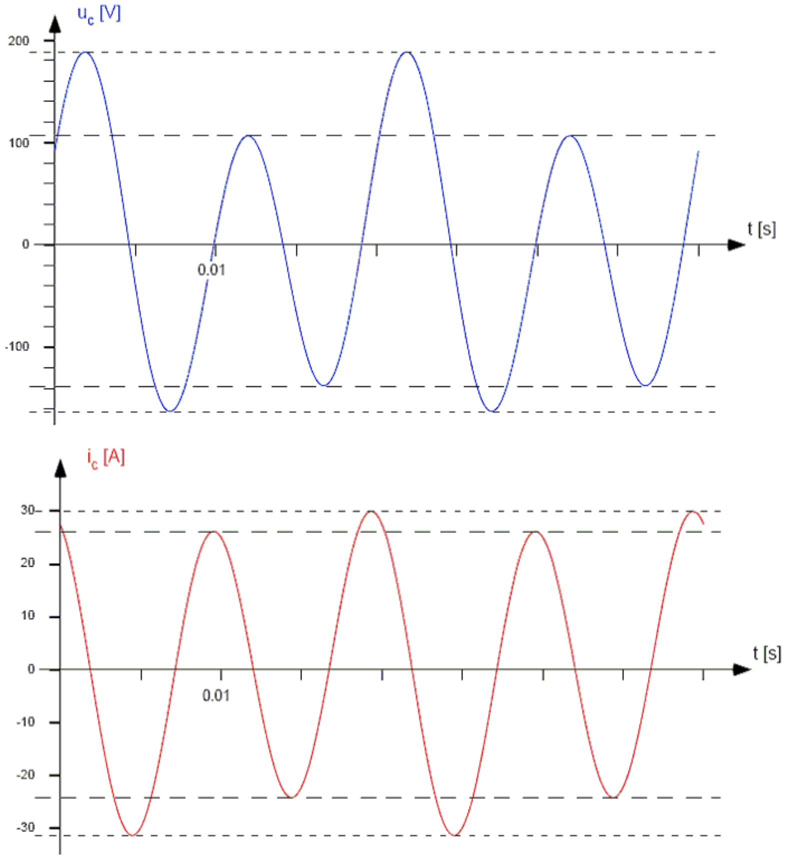
$\Big\}\; u_c(t) = u_{c1}(t) + u_{c2}(t)$

$$i_{c1}(t) = |\,\underline{I}_{C1}\,| \sqrt{2} \cos(\omega_1 \cdot t + \arg(\underline{I}_{C1})) = \underline{4{,}04 \cos(314{,}2 \cdot t - 1{,}879)\text{ A}}$$
$$i_{c2}(t) = |\,\underline{I}_{C2}\,| \sqrt{2} \cos(\omega_2 \cdot t + \arg(\underline{I}_{C2})) = \underline{27{,}89 \cos(628{,}3 \cdot t + 0{,}358)\text{ A}}$$

$\Big\}\; i_c(t) = i_{c1}(t) + i_{c2}(t)$

Graphically, we obtain the results below:




u_c [V]
200
100
0
-100
0.01
t [s]
i_c [A]
30
20
10
0
-10
-20
-30
0.01
t [s]